AN ELLIPSOID SHAPE APPROXIMATION APPROACH FOR 3D SHAPE REPRESENTATION

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ABSTRACT

Shape representation plays an important role to preserve shape geometry information of any object. The majority of existing shape representation methods used sphere to approximate 3D object. Sphere is divergence in nature and hence the mapping between sphere and object is non-compact and the resultant shape representation becomes less informative. In this paper, we propose a shape representation method based on ellipsoid to approximate 3D object. Ellipsoid is convergence in nature and it is better fitted to the body of the shape rather than sphere. The proposed shape representation method using ellipsoid is able to generate informative shape sampling function capturing more detail information smoothly of an object. Our proposed method decreases the approximation error than the conventional spherical based method.

1. INTRODUCTION

Human vision can perceive and define any shapes, objects or images especially three dimensional (3D) world using eyes and brain. Human learns to recognize the world from beginning of life acquiring knowledge of the world. In computer vision, it is very difficult to recognize objects as humans. The learning process to acquire knowledge may be more useful in the case of video surveillance system, robots and etc. Shape is one of the most useful and important properties to describe an object. Shape visualization of an object is one of the most important tasks and it is tolerable to humans, even parts of shape are not visible. Thus a challenging issue in computer vision is an efficient shape representation. Needless to say, we live in a 3D world. The recent development in camera technology such as 3D scanner and stereo vision leads to increase datasets of 3D model worldwide for 3D shape representation & recognition and gets greater attention for research. 3D shape representation and recognition can benefit many applications such as 3D search engine, robotics, biometric authentication, CAD/CAM industry, medical imaging, molecular biology and computer vision.

This paper aims to propose a 3D shape representation method which can be applied to 3D object recognition purpose in a large scale scenario.

The goal of this research is to create a mapping between human and machine vision by considering the efficiency of 3D shape representation to store their attribute in a highly visible manner. In this paper, we propose an efficient ellipsoid based shape representation method approximating the 3D object into an ellipsoid. The approximation error measured using Hausdroff distance using Princeton Shape Benchmark (PSB) [1] shows less error than other conventional shape representation method based on the sphere which is used for generic 3D object recognition.

The rest of the paper is organized as follows. In section 2, we present a brief review on existing shape representation and also state their limitations. In section 3, we explain the problem of spherical shape representation method used in the existing methods with examples of 3D shapes. Section 4 introduces a new method for 3D shape representation. In section 5, we provide the comparison results of the proposed method against the existing method measuring Hausdroff based shape approximation error. Finally, we conclude the paper in section 6.

2. LITERATURE REVIEW

In this section, we provide a brief review of existing shape representation methods which is employed for 3D object recognition purpose. Survey paper can find this topic in [2], [3], [4]. According to Loncaric [3], shape representation schemes are used to preserve important characteristics. Shape representation schemes are useful since it visualizes the shape accordingly.

Conventionally there are two categories for shape representation: local and global methods. Local methods attempt to represent objects as a set of faces or edges such as distance between faces or the angle between faces normal. Early local methods are used to handle polyhedral objects but not always suitable for curved objects including [5] and [6]. Vranic and Saupe [7] also presented an approach for 3D model retrieval by finding the number of polygons, vertices, surface area and etc. Earlier local methods were limited to preserving the object’s attribute where the features are extracted directly. Moreover local methods are noise sensitive and limited to their polygon connectivity.

Global methods are meant for mapping an object to a specified coordinate system in a predefined domain as an implicit function such as spherical function. Researchers such as Saupe & Vranic [8], Funkhouser et al. [9], Kazhdan et al. [10], Laga et al. [11] and Lmaati et al. [12] used sphere as shape primitive to represent 3D shape.
Afterwards, they employed spherical shape function in various ways such as, spherical harmonics, spherical wavelets and spherical helices which were later used for generic 3D object recognition purpose.

Although the shape function mapping methods on a sphere are popular means, they are limited to ‘fitting’ an object. Sphere is divergence in their nature and thus the representation is not closure to the body of the shape. Moreover, Kajya [13] mentioned that the spherical surface does not contain snugly. This cause may lead to non-uniform shape representation and increases the shape approximation error.

In the literature there are few techniques which used multiple numbers of primitives such as [14, 15] as a tree structure shape function for shape representation. Although those type of methods decrease approximation error but the complexity is also increases due to the requirements of number of primitives. Those methods may not be suitable to retrieve 3D object from large number of database. Moreover, extra collision handling method [16] is also requires to avoid the collision among the primitives.

3. NON-SMOOTH SAMPLING PROBLEM ON SPHERICAL SHAPE FUNCTION

The common and popular shape function is defined on the unit sphere measured on angles $\theta$, $0<\theta<\pi$ and $\phi$, $0<\phi<2\pi$. The spherical extent function (EXT) [8] $f(\theta, \phi)$ is regarded as the shape representation function. Commonly Ray casting method [17] is used to sample the object by casting rays from the object’s centre in the radial direction $(\theta, \phi)$. A Ray $R(t)$ is defined as

$$R(t) = O + tD$$  (1)

Where $O$ is the origin of the ray and $D$ is the direction. Rays are passed in the radial direction and ray-triangle intersection points are computed. Triangle barycentric coordinates [18] are used to determine whether the intersection point belongs to inside the triangle. If there is more than one intersection points the farthest point is taken into account and if intersection does not occur then centre of mass is regarded as the point. Shape function is computed by obtaining the points at the different location. Here, we analyzed the spherical shape representation function using the grid size 64*64 which is a popular representation. Figure 1 shows an illustration of a spherical based representation of examples of airplane and shark models. The models are collected from PSB.

The non-uniform problem of spherical shape representation illustrated in Figure 1 may be occurring due to non compact fitting of the sphere to the object. Sphere is divergence in nature and hence it does not approximate well to the body of the shape. From the observation it is obvious that the spherical sampling does not provide a smooth sampling of the airplane and shark model which is shown in Figure 1. The resultant spherical sample shows that head, fins and tail parts are poorly sampled. The term 'poorly' means that the sampling has not occurred properly where the original shape information can not extract well. The non-uniform shape function means that central part of the model is well sampled whereas outmost part of the object provided poorly sampled. Hence this representation based on sphere provides less informative representation. This type of sampling causes loss of characteristics information which will not be able to provide informative representation of any 3D model for feature extraction and recognition purpose. This may also lead to increase approximation error.

4. ELLIPSOID BASED SHAPE REPRESENTATION

To avoid the aforementioned problem explained in section 3, we propose an ellipsoid based compact representation to generate an informative and uniform shape function of 3D object. Ellipsoid is convergence in nature and better fitted to approximate the object. The proposed method using ellipsoid can be used to employ with enriched shape information to lower the shape approximation error. Equation 2 shows the definition of an ellipsoid and the coordinate function can be used to generate an ellipsoid. In the Cartesian coordinate system an ellipsoid can be represented by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$  (2)

Once the ellipsoid surface is computed ray-triangle intersection algorithm [18] is applied and the set of corresponding points on the surface are obtained $f(\theta, \phi)$, $E_c = [X_{(\theta, \phi)} Y_{(\theta, \phi)} Z_{(\theta, \phi)}]$. The ellipsoid is computed using the same grid size 64*64 providing 4096 sampled points as the spherical representation which was shown before in Figure 1. In Figure 2 we show ellipsoid based reconstructed objects. PCA [19] is used to normalize 3D models and the centre of the model is computed as follows:

Consider that there is a set of vertices $V$ of a model where,

$$V = \{V_1, V_2, ..., V_n\}$$

$$V_i = [x_i, y_i, z_i] \in S^3$$  (3)

So the centre point $V_c(x_c, y_c, z_c)$ is defined as

$$x_c = \frac{1}{n} \sum_{i=1}^{n} x_i,$$

$$y_c = \frac{1}{n} \sum_{i=1}^{n} y_i,$$

$$z_c = \frac{1}{n} \sum_{i=1}^{n} z_i$$  (4)

Sampling with more points may improve the quality of approximation. Below three reasons are discussed for choosing 64 as the used sampled dimension:

(i) Firstly, it is observed that, if the shape is sampled using 32 then the sampling provides a poor
approximation for an example of an ant model using the ellipsoid approximated shape function illustrated in Figure 3 (a). It is also noticed that shape sampling with 64 and 128 shown in the Figure 3 (b) & 3 (c) respectively did not show significant difference in the ant model which will make us to consider increasing the sampling rate. Increasing sampling rate may provide a better solution for more complex objects.

(ii) Secondly, space complexity is higher, if the sampling rate will be regarded as 128. Table I summarized the comparison of dimensions of 32, 64 and 128.

(iii) Thirdly, time complexity also increases when the size of the vertices of models increases. For example a car model containing 308 numbers of vertices took 5.9 seconds to sample the object when the sampling rates were 64. The computation time took 22.2 seconds, if the sample size were selected as 128 for the same model.

The proposed method not only describes shape better but also minimizes the size of the descriptor. Although the proposed shape representation method minimizes the dimension, the sampling provides informative shape function than the spherical based mapping function. In addition, the dimension of the descriptor will also minimize if the sampling dimension minimize. In general, ray casting method is slow and the time complexity also increases using ellipsoid since the sampling requires extra operation due to semimajor and semiminor axis of ellipsoid. Still the performance is not degraded.

Table I. Comparisons of various sampling sizes

<table>
<thead>
<tr>
<th>Sampling rate (E)</th>
<th>Sampling points (E^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>1024</td>
</tr>
<tr>
<td>64</td>
<td>4096</td>
</tr>
<tr>
<td>128</td>
<td>16384</td>
</tr>
</tbody>
</table>

Fig. 1 Spherical representation causes non-smooth and non-uniform approximation

Fig. 2 Proposed compact shape representation based on ellipsoid
5. APPROXIMATION ERROR MEASUREMENT

We used Hausdorff distance [20] to measure the approximation error and compared with spherical based mapping. Hausdorff distance has been used for measuring surfaces error in many applications such as mesh simplification, geometry compression, surface distortion and etc [16]. Hausdorff distance between the original surface X and approximated surface Y can be defined as:

\[ d(X,Y) = \max [d(X,Y),d(Y,X)] \]  (5)

The comparison of approximation error of the proposed ellipsoid based shape representation against spherical representation with the examples of various models are shown in Figure 4. Figure 4 shows the comparison of approximation error of different fighter\_jet, hand\_gun, fish and bird models which are collected from Princeton Shape Benchmark [1]. Those models are chosen randomly from the database. A class contains various types of models for example, fighter\_jet class contains different types of fighter\_jet models with different orientation etc. In Figure 4 (a), the horizontal axis represents various fighter\_jet models from fighter\_jet class which is taken randomly from Princeton Shape Benchmark, and so on for the Figures 4 (b, c & d).

From the observation, the shape approximation error decreases using the proposed ellipsoid representation method. For example, Figure 4 (a) shows that the error is as low as 30% of a fighter model using ellipsoidal rather than spherical representation. It is to believe that due to the compact representation of ellipsoid to the shape, the approximation error is smaller than the existing sphere based approximation.

6. CONCLUSION

In this paper, we propose an efficient uniform and compact ellipsoid based 3D shape representation method. This method provides an informative representation which is very useful and efficient for 3D shape descriptor extraction purpose in general kinds of shape. The non-smooth sampling causes in the case of using sphere is avoided by introducing the idea of proposed ellipsoid shape representation. In addition, the proposed method lowers the approximation error based on Hausdorff distance.

7. REFERENCES


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**Fig. 4** Comparison of approximation errors of the proposed ellipsoid shape representation scheme against spherical representation of various (a) fighter_jet, (b) hand_gun, (c) fish & (d) bird models.