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DIFFRACTED ELECTROMAGNETIC WAVE IN FRESNEL REGION OF THE PYRAMIDAL HORN ANTENNA

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ABSTRACT

In the microwave band, the radiated wave from the pyramidal horn antenna is calculated in the diffracted field by the Fresnel approximation. In addition, the Fresnel approximation has been introduced into the diffracted field with half infinite diffraction plane.

In this report, the electromagnetic diffracted field in the pyramid horn antenna is calculated under the Fresnel approximation. As a result, the complex hologram is made by diffracted electromagnetic wave with the analytical expression. Also, the diffracted electromagnetic wave is calculated with fast Fourier transform(FFT). Moreover, the wave source is numerically reconstructed as a result of inverse fast Fourier transform(IFFT) on the horn aperture.

1. INTRODUCTION

When the electromagnetic field distribution of the aperture antenna is measured directly, it is known well not to be able to measure the electromagnetic field distribution on an aperture plane by the interaction of the receiving probe.

Measurement is difficult from the restrictions in measuring space, and far field is measuring the near field, decides about far field and is doing from before[1]. Measurement system using electro–optic sensor is contrived to improve the resolution in measurement in this near field[2].

For instance, the aperture antenna like the pyramid horn antenna approximately calculates the radiation field from the electromagnetic field distribution assumed on the aperture plane[3]. More strictly, a highly precise numerical analysis of moments method is used[4]. It's considered to use a high frequency electromagnetic field simulator present[5]. The analytical calculation is performed based on the Huygens–Fresnel principle, and is expressed for the space distribution using the Fresnel function. The Fresnel function has been introduced into the diffracted field with half infinite plane[6, 7, 8].

This manuscripts report concretely a simple expression which succeeded in our inducing by applying the Fresnel approximation [8]. Such electric field distribution is analytically and numerically calculated in Fresnel region, and the purpose of the present study makes the complex hologram[9, 10].

Moreover, the wave source is numerically reconstructed as a result of inverse fast Fourier transform on the horn aperture form the electromagnetic field distribution in the Fresnel region.

2. FORMULATION

2.1. THE DIFFRACTED WAVE

In Fig.1, the pyramidal horn antenna and geometry of the source rectangular coordinate (x', y') are shown. The horn antenna shape is assumed that a_1 is the width in the x' direction and b_1 is the height in the y' direction. We consider Huygens' construction when the electric field distribution (TE$_{10}$ mode) in the waveguide of the rectangular section expands on the sphere here at the aperture of the horn antenna[3]. The electric field distribution on this aperture is given by the next expression as

$$E_y(x', y') = \cos\left(\frac{\pi}{a_1}x'\right) e^{-jk\left(x'^2/\rho_1 + y'^2/\rho_2\right)/2} \tag{1}$$

where the existing area in the electric field is in the range of $|x'| \leq a_1/2$ and $|y'| \leq b_1/2$ and $k = 2\pi/\lambda$, λ is wave number. The time dependence is $e^{j\omega t}$, and the description is omitted in this report thereafter.

When coordinates on the observation plane where the origin was matched to coordinates in the source of wave are considered at the position, the diffracted field is obtained the Fresnel–Kirchhoff integral is performed based on the

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Huygens–Fresnel principle, as follows[6]

$$E_y\left(x,y;z\right)=\frac{jk}{4\pi}\iint_{-\infty}^{\infty}E_y(x',y')p(x-x',y-y';z)dx'dy' \quad (2)$$

where

$$p(x-x',y-y';z)=\left(1-\frac{1}{jk}\frac{\partial}{\partial z}\right)\frac{e^{-jkR'}}{R'} \quad (3)$$

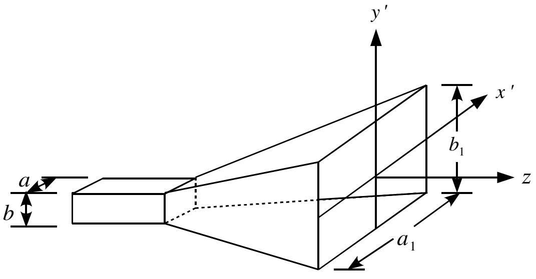
and

$$R'=\left\{z^2+(x-x')^2+(y-y')^2\right\}^{1/2} \quad (4)$$

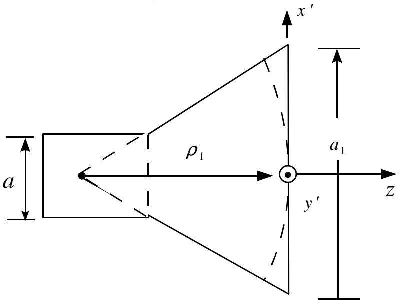
Substituting in Eq.(2) with Eq.(1) and partial differentiation by z of Eq.(3), that is normalized by the wavelength, became following

$$E_y\left(x,y;z\right)=\frac{jE_0}{2}\int_{-b_1/2}^{b_1/2}\int_{-a_1/2}^{a_1/2}\cos\left(\frac{\pi}{a_1}x'\right)e^{-j\pi(x'^2/\rho_1+y'^2/\rho_2)}\frac{e^{-j2\pi R'}}{R'}\left\{1+\left(1+\frac{1}{j2\pi R'}\right)\frac{z}{R'}\right\}dx'dy' \quad (5)$$

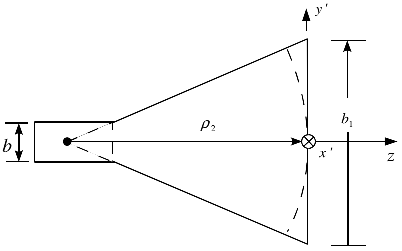
An above equation is two-dimensional integration for wave source (x',y') that is evaluated with numerical value.



(a) Pylamidal horn



(b) Side (H–plane) view



(c) Side (E–plane) view

Figure 1: The geometrical coordinate and the pyramidal horn antenna

2.2. ELECTROMAGNETIC FIELDS ON APERTURE

The parameter of a pyramidal horn antenna in after numerical value calculation is made the street indicated in table 1. Those parameters are apllied in Eq.1, and we can obtained the electromagnetic fields on aperture (z=0) as shown in Fig.2.

Table 1: The parameters of pyramid horn antenna

	a_1	b_1	ρ_1	ρ_2
horn A	2.976	2.160	5.553	5.556
horn B	4.816	3.568	6.616	6.222

2.3. DIFFRACTED WAVE ON NORMAL DIRECTION

When $x=0$ and $y=0$ in Eq.(5), those integrated variable are (x,y) without observation point for diffracted wave on z axis as follows

$$E_y\left(z\right)=\frac{ja_1b_1}{2}\int_0^1\int_0^1E_y(x,y)\frac{e^{-j2\pi r}}{r}\left\{1+\left(1+\frac{1}{j2\pi r}\right)\frac{z}{r}\right\}dxdy \quad (6)$$

where $r=\left\{z^2+(a_1x/2)^2+(b_1y/2)^2\right\}^{1/2}$, $E_y(x,y)=E_0\cos\left(\frac{\pi}{2}x\right)e^{-j\pi\left\{(a_1x/2)^2/\rho_1+(b_1y/2)^2/\rho_2\right\}}$.

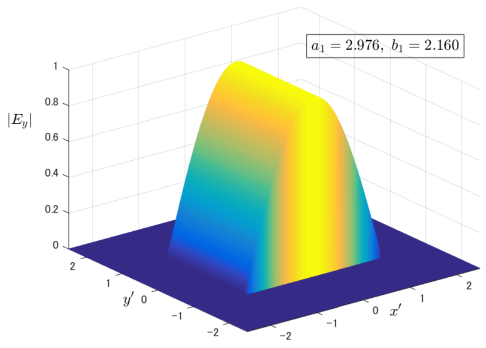
In Fresnel region,

$$E_y\left(z\right)\cong ja_1b_1\frac{e^{-j2\pi z}}{z}\int_0^1\int_0^1E_y(x,y)e^{-j\pi\left\{(a_1x/2)^2/z+(b_1y/2)^2/z\right\}}dxdy \quad (7)$$

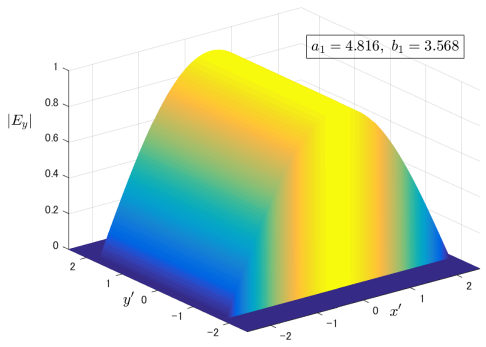
and in Fraunhofer region,

$$E_y\left(z\right)\cong ja_1b_1\frac{e^{-j2\pi z}}{z}\int_0^1\int_0^1E_y(x,y)dxdy \quad (8)$$

In expression Eqs.(6)∼(8), a variable separation is possible

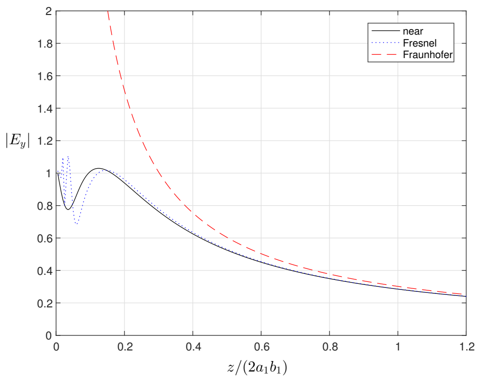


(a) horn A

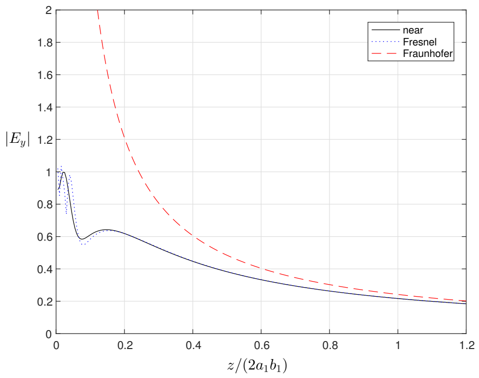


(b) horn B

Figure 2: Electromagnetic fields on aperture



(a) horn A



(b) horn B

Figure 3: Diffracted wave on z axis

by the Eqs. besides the Eq.(6), but two-dimensional numerical value integration is performed for comparison.

As shown in Fig.3, the calculation results are expressed on z axis, the near-field region and the Fresnel region be indicated resembles until the wave source. In the case of horn A, $z_1 = 12.86$ is meaning $z_1/(a_1b_1) \cong 1.0$, this plane of observation is located on Fresnel region. And the case of horn B, $z_1 = 34.37$ is meaning $z_1/(2a_1b_1) \cong 1.0$, this plane of observation is also located on Fresnel region. These observation points are the location which becomes Fraunhofer region from here.

3. DIFFRACTED WAVE IN FRESNEL REGION

3.1. ANALYTICAL EXPRESSION WITH FRESNEL FUNCTION

Diffracted field is shown analytically using Fresnel function at Fresnel region so that it may be indicated by appendix A[7]. The result which calculated diffracted field are obtained such as Figs.4 and 5 using a parameter in table 1.

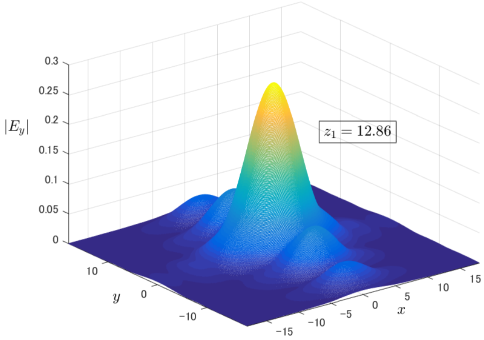
3.2. NUMERICAL CALCULATION WITH FFT

It's also possible to calculate and get diffracted field numerically using FFT. The result calculated using a parameter in same table 1 are indicated on Figs.6 and 7.

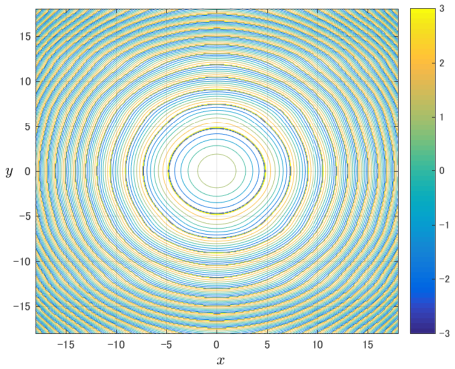
Since putting it in the calculation of diffracted field by these FFT, you don't always have to give an electromagnetic field of aperture plane analytically such as Eq.(1). For example, the amplitude and phase of the numerical analysis electromagnetic field and the distribution which is the field by the moments method.

4. RECONSTRUCTION FIELD ON APERTURE

The example which reconstructed electric field distribution in aperture plane of a wave source numerically using IFFT was indicated on Fig.8 from the diffracted field obtained analytically.

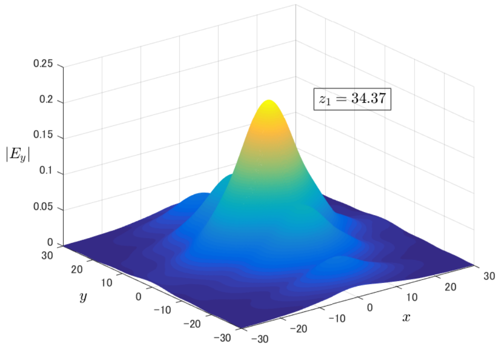


(a) Amplitude (horn A)

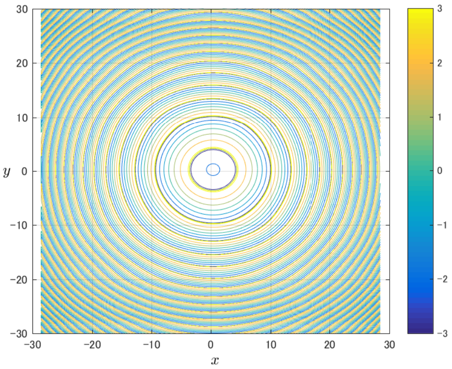


(b) Phase (horn A)

Figure 4: Diffracted wave at $z_1 = 12.86$ (analitical)



(a) Amplitude (horn B)



(b) Phase (horn B)

Figure 5: Diffracted wave at $z_1 = 34.37$ (analitical)

5. CONCLUSION

The diffracted electromagnetic wave in the pyramidal horn antenna is calculated analytical expression with the Fresnel approximation. And, the diffracted electromagnetic wave also is computed numerical evalution with FFT on the computer. Moreover, the image that this patterns are reconstructed numerically is obtained with IFFT. We'll be by base in a convolution theorem in the near future, it's expected to study a calculation method of the diffracted field by spatial spectrum.

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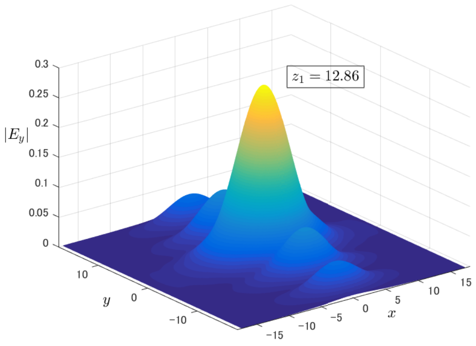
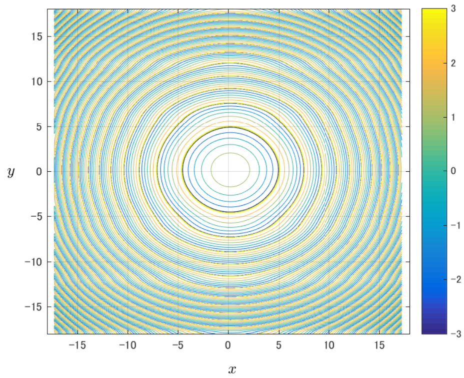
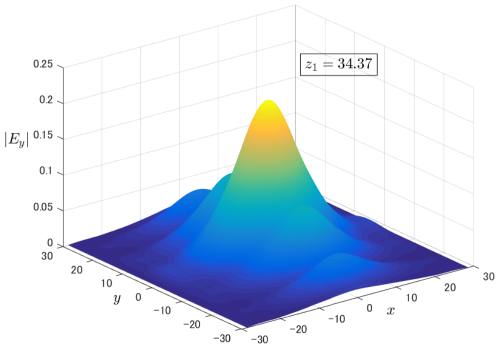


(a) Amplitude (horn A)

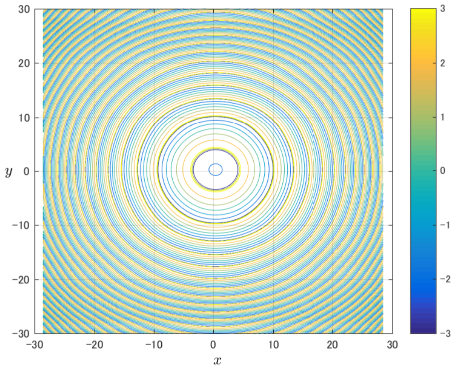


(b) Phase (horn A)

Figure 6: Diffracted wave at $z_1 = 12.86$ (numerical)



(a) Amplitude (horn B)



(b) Phase (horn B)

Figure 7: Diffracted wave at $z_1 = 34.37$ (numerical)

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APPENDIX

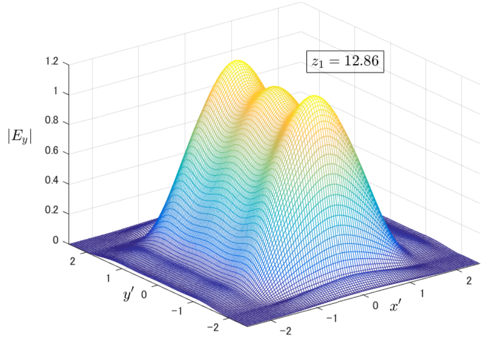
A. THE INTEGRATION OF FRESNEL DIFFRACTED WAVE

From Fresnel’s approximation of Eq.(5) in $z = z_1$, we have following

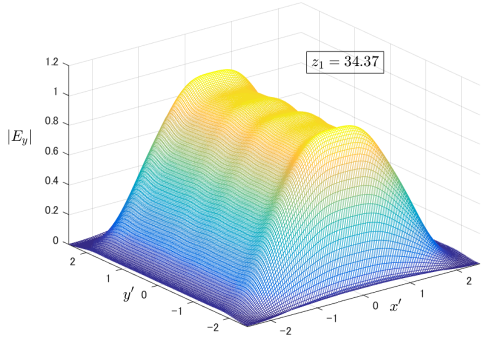
$$\begin{aligned} E_y\left(x, y; z_1\right) &= jE_0 \frac{e^{-j2\pi z_1}}{z_1} \\ &\int_{-b_1/2}^{b_1/2} \int_{-a_1/2}^{a_1/2} \cos\left(\frac{\pi}{a_1} x'\right) e^{-j\pi(x'^2/\rho_1 + y'^2/\rho_2)} \\ &e^{-j\pi\{(x-x')^2 + (y-y')^2\}/z_1} dx' dy' \\ &= jE_0 \frac{e^{-j2\pi z_1}}{z_1} \cdot I(x, y) \qquad \text{(A.1)} \end{aligned}$$

A result of the integration can do the separation of variables in the above equation at this stage, and is asked as follows

$$\begin{aligned} I(x, y) &= I_x(x) \cdot I_y(y) \\ & e^{-j\pi x^2/z_1} I_1(x) \cdot e^{-j\pi y^2/z_1} I_2(y) \quad \text{(A.2)} \end{aligned}$$



(a) horn A



(b) horn B

Figure 8: Reconstruction field on aperture

$$\begin{aligned} I_1(x) &= \frac{1}{4}\sqrt{\frac{2}{K_{11}}}e^{j\pi\left(\frac{1}{2a_1}+\frac{x}{z_1}\right)^2/K_{11}} \\ &\quad \left\{F_{res}\left[\sqrt{\frac{2}{K_{11}}}\left(\frac{K_{11}a_1}{2}-\frac{1}{2a_1}-\frac{x}{z_1}\right)\right]\right. \\ &\quad \left.+F_{res}\left[\sqrt{\frac{2}{K_{11}}}\left(\frac{K_{11}a_1}{2}+\frac{1}{2a_1}+\frac{x}{z_1}\right)\right]\right\} \\ &+ \frac{1}{4}\sqrt{\frac{2}{K_{11}}}e^{j\pi\left(\frac{1}{2a_1}-\frac{x}{z_1}\right)^2/K_{11}} \\ &\quad \left\{F_{res}\left[\sqrt{\frac{2}{K_{11}}}\left(\frac{K_{11}a_1}{2}-\frac{1}{2a_1}+\frac{x}{z_1}\right)\right]\right. \\ &\quad \left.+F_{res}\left[\sqrt{\frac{2}{K_{11}}}\left(\frac{K_{11}a_1}{2}+\frac{1}{2a_1}-\frac{x}{z_1}\right)\right]\right\} \end{aligned} \tag{A.3}$$

where $K_{11} = \dfrac{1}{\rho_1} + \dfrac{1}{z_1}$

$$\begin{aligned} I_2(y) &= \frac{1}{2}\sqrt{\frac{2}{K_{21}}}e^{j\pi\left(\frac{y}{z_1}\right)^2/K_{21}} \\ &\quad \left\{F_{res}\left[\sqrt{\frac{2}{K_{21}}}\left(\frac{K_{21}b_1}{2}-\frac{y}{z_1}\right)\right]\right. \\ &\quad \left.+F_{res}\left[\sqrt{\frac{2}{K_{21}}}\left(\frac{K_{21}b_1}{2}+\frac{y}{z_1}\right)\right]\right\} \end{aligned} \tag{A.4}$$

where $K_{21} = \dfrac{1}{\rho_2} + \dfrac{1}{z_1}$

B. INDEFINITE INTEGRAL FORMULA

$$\begin{aligned} &\int e^{-j(Ax^2+2Bx)}dx \\ &= \sqrt{\frac{\pi}{2A}}e^{jB^2/A}\cdot F_{res}\left[\sqrt{\frac{2}{\pi A}}(Ax+B)\right] \\ &= \sqrt{\frac{\pi}{2A}}e^{jB^2/A}\cdot\left\{C\left[\sqrt{\frac{2}{\pi A}}(Ax+B)\right]\right. \\ &\qquad \left.-jS\left[\sqrt{\frac{2}{\pi A}}(Ax+B)\right]\right\} \end{aligned} \tag{B.1}$$

where

$$F_{res}(x) = \int_0^x e^{-j\frac{\pi}{2}t^2}dt = C(x) - jS(x) \tag{B.2}$$

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt \tag{B.3}$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt \tag{B.4}$$