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A SIMPLE EVAPORATION MODEL OF WATER DROPLETS BASED ON SPH METHOD BY CONSIDERING ATMOSPHERIC HEAT TRANSFER

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ABSTRACT

We have proposed a model to depict water droplets in computer graphics by using the smoothed particle hydrodynamics (SPH) method. In this paper, we propose a simple model for the evaporation of water droplets based on an extended SPH method by considering the heat transfer from air and surface.

1. INTRODUCTION

Depiction of natural phenomena by computer graphics (CG) is used in such as video work and video game. Smoothed Particle Hydrodynamics (SPH) method [1] which is one of the particle method is often used to depict a fluid. It has been well studied the large scale fluid simulation such as the water that is poured into the cup and wave generation in the sea. On the other hand, in the small scale fluid like a water droplet, we must consider and depict natural phenomena such as interfacial tension and contact angle that were able to ignore in the large scale.

Abe et al. [2] devised a water droplet model to depict it in CG. This model can depict contact angle produced by interfacial tension of water droplet by using SPH method. Sato et al. [3] devised a water droplet model can depict dynamic contact angle which appear when droplet slide down on the slope.

However, these two methods are water droplet geometric modeling technology that assuming keeping a liquid state. These cannot depict appearance of the water droplet evaporate because these are not considered changes that phase transition from a liquid to a gas with time.

In this paper, we aim to build a simple model that can depict the evaporation of water droplets. In our method, setting a temperature parameter to particles of SPH method, to calculate heat transfer between air-particle, solid surface-particle, and particle-other particles. The particle's temperature parameter rises, and if it exceeds the threshold, erase the particle.

2. CONVENTIONAL METHOD

2.1. Interfacial Tension Model by Abe et al.

The line that solid, liquid, and gas are in contact simultaneously, called contact line (Fig. 1). The angle between the liquid surface and the solid surface at contact line is contact angle (angle θ at Fig. 1). γ_s, γ_l, γ_{sl} is solid interfacial tension, the liquid interfacial tension, and the interfacial tension between solid and liquid [4].

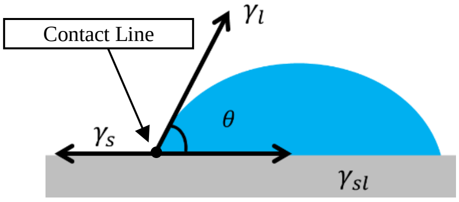


Fig. 1 Contact Angle

Abe et al. [2] build a water droplet model in SPH method that can depict contact angle produced by interfacial tension. They refer to the particles that distance from a particle i is within the threshold as "neighbor particle." The number of neighbor particles referred to in N_i, and the numbers in circle in Fig. 2 represents N_i. As shown in Fig. 2, the more the particles of droplet are inside of the droplet, the larger the number of neighbor particles N_i. But the particles of droplet are outside, the smaller the number of neighbor particles N_i. Therefore, the force that toward the water droplet inside as inversely proportional to the number of neighbor particles N_i, instead of the interfacial tension, it is decided to represent the contact angle of water droplet.

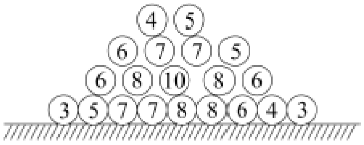


Fig. 2 Example of the number of neighbor particles[2]

Specifically, multiplied by a constant c to adjust the strength of the interfacial tension to the reciprocal of N_i is interfacial tension $\overrightarrow{F_i}$. $\overrightarrow{F_i}$ is indicated by the following equation, $\vec{x}_{ij}$ is a unit vector in the direction from the particle i to neighbor particle j.

$$\vec{F_i} = \frac{c}{N_i}\vec{x}_{ij} \qquad (1)$$

Further, when the volume of the water droplet is sufficiently small, the gravity does not affect water drop shape, the influence of interfacial tension is dominant. At this time, the water droplets have a shape close to a sphere. When the volume of the water droplets is large, its radius is also increased concurrently, water droplets under the influence of gravity becomes flat shape [4].

Thus, Abe et al. [2] zeroed interfacial tension of the particle when N_i is more than the threshold. By providing the interfacial tension only on the particle in the vicinity of the interface with the solid surface, even allowed water droplets representation of flat shape.

By changing the value of the constant c for adjusting the intensity of the interfacial tension of the equation (1), different water droplet of contact angles can be depicted.

2.2. Dynamic Contact Angle Model by Sato et al.

Sato et al. [3] more extended Abe's method, and proposed a water droplet geometric model which can depict dynamic contact angles that appear when droplets move on solid surface (Fig. 3).

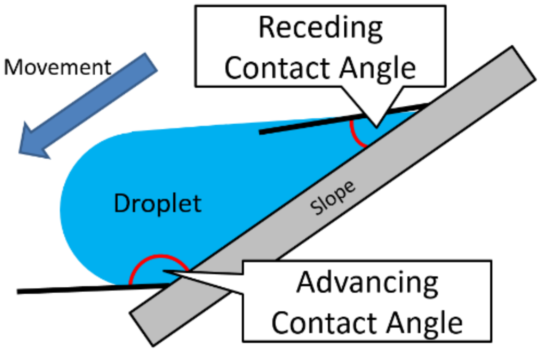


Fig. 3 Dynamic contact angle

On the particle i interfacial tension is working, consider the drag $\vec{R_i}$ of the opposite direction in the direction of movement. Determine the dot product of velocity unit vector $\vec{v_i}$ of particle i and unit vector $\vec{f_i}$ of interfacial tension $\vec{F_i}$ obtained by Abe's method [2]. The coefficient multiplied by the constant k_r in the absolute value of the dot product. As in the following equation, drag $\vec{R_i}$ is determined by scalar multiplying this coefficient to unit inverse vector $-\vec{v_i}$ of the direction of particle movement.

$$\vec{R_i} = k_r|\vec{v_i}\cdot\vec{f_i}|\cdot(-\vec{v_i}) \qquad (2)$$

Fig. 4 is focused on the particle i working interfacial tension, and shows the relationship between the direction vectors. Particle i is located at the more front or the more rear in the water droplet, the absolute value of the dot product of $\vec{v_i}$ and $\vec{F_i}$ is the larger, and so drag $\vec{R_i}$ increases. Conversely, when particle i is flanked to the moving direction of the water droplet, the dot product value becomes smaller, the influence of the drag $\vec{R_i}$ is small. By introducing the drag $\vec{R_i}$, allowed the expression of the droplet shape in consideration of the dynamic contact angle.

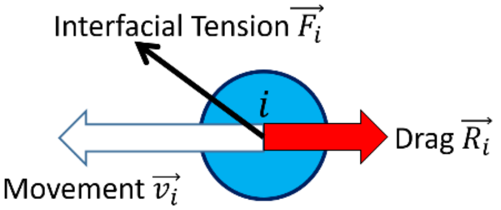
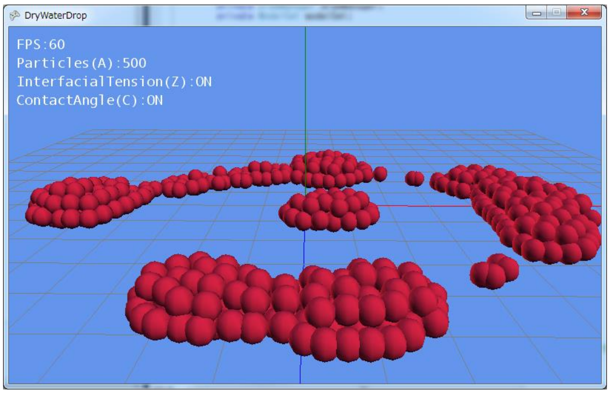


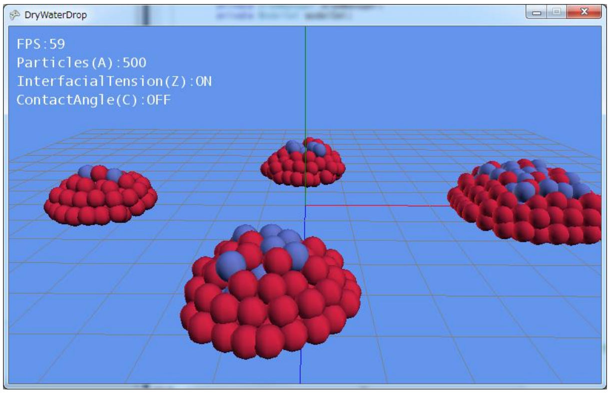
Fig. 4 Exerted forces on particle *i*

Fig. 5 shows the one to which Sato's method is applied and the other not. It shows that Sato's model expresses the characteristic of dynamic contact angle. We use Sato's model and extend it.



FPS:60
Particles(A):500
InterfacialTension(Z):ON
ContactAngle(C):ON

(a) Sato's method



FPS:59
Particles(A):500
InterfacialTension(Z):ON
ContactAngle(C):OFF

(b) Abe's method

Fig. 5 Presence or absence of dynamic contact angle model

3. PROPOSED METHOD

Water droplet geometric models [2][3] described in the previous section are state of liquid is maintained. These cannot depict the change and deformation with the phase transition of the first kind such as the evaporation of water droplets.

In our method, as the shape deformation model of water droplets, based on the Sato's model [3]. We propose a model can depict water droplets evaporating. Calculating the heat transfer between air and the particles in the vicinity of the interface, between a solid surface and the particles in contact with the solid surface, and between the particles. By realizing a mechanism to erase when the particles reached a certain temperature, it represented evaporation.

3.1. Calculation of the Heat Transfer Amount

Define t_a as the temperature of the air, t_s as the temperature of the solid surface, t_i as the temperature of the particle i, t_j as the temperature of the other particles j in contact with the particle i. Heat transfer between two objects or its environment that is following equation (3) [5]. Heat transfer amount Q is proportional to the area A of the surface perpendicular to the heat flow and the difference of temperature T of object a and b. h is heat transfer coefficient.

$$Q = hA\,(T_a - T_b) \tag{3}$$

In this paper, we calculate it based on Equation (3), considering the number of particles. Here, the area A of the surface perpendicular to the heat flow is assumed to be constant in between the particles. We limit the subjects where heat transfer occurs to the following three.

(1) The heat transfer amount Q_{ia} from air, to the particles which are exposed to the air (Fig. 6Fig. 6).
(2) The heat transfer amount Q_{is} from solid surface, to the particles which contact the solid surface (Fig. 7).
(3) The heat transfer amount $Q_{ij}\,(i \neq j)$ from the particles j neighboring to the particle i (Fig. 8).

Summing the three heat transfer amount above. Regard the sum as the heat transfer amount Q_i to the particle i, add to the temperature t_i of the particle i. Describe below how to obtain the amount of heat transfer above (1), (2), and (3).

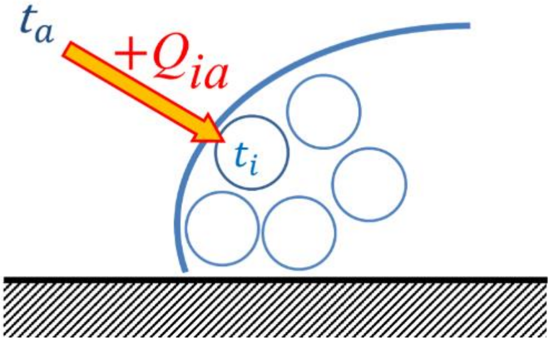


Fig. 6 Heat transfer from air

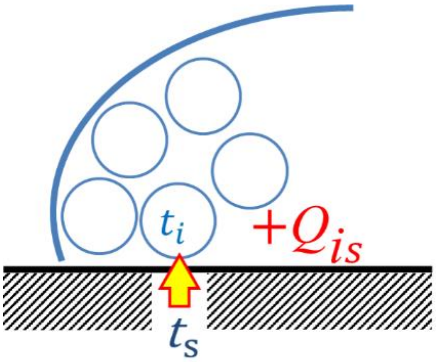


Fig. 7 Heat transfer from solid surface

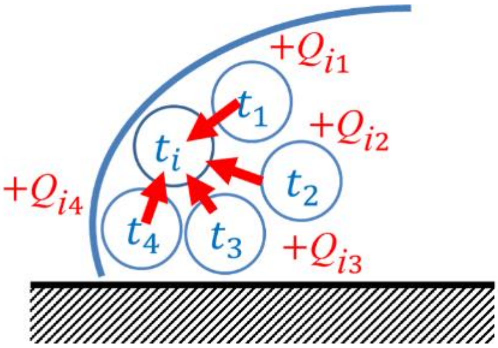


Fig. 8 Heat transfer from neighbor particles

3.1.1. Heat Transfer from Air

The number of neighbor particle N_i of the particle i is less the threshold N, this particle i is assumed to exposed to air. Conversely, the number of neighbor particle is more than the threshold N, this particle i is not assumed to exposed to air. Define k as thermal conductivity.

If the particle i is exposed to air, calculate the heat transfer amount Q_{ia} from the air to the particle. The number of neighbor particle N_i is the smaller, an area exposed to the air is the larger, easy to heat is transferred. Therefore, by multiplying by the reciprocal of N_i, it is taken as the area. Let dt be a time step.

$$Q_{ia} = \frac{k}{N_i}(t_a - t_i)\,dt \tag{4}$$

Incidentally, if it is determined not to exposed to the air $(N_i > N)$, consider $Q_{ia} = 0$.

3.1.2. Heat Transfer from Solid Surface

Perform collision detection of the particle and the solid surface, with respect to the particle i contacting the solid surface, the heat transfer amount Q_{is} from solid surface can be calculated by the following equation.

$$Q_{is} = l(t_s - t_i)\,dt \tag{5}$$

Where l is the thermal conductivity of the solid surface, it is $0 < l < 1$.

3.1.3. Heat Transfer between Particles

The heat transfer amount Q_{ij} from the particles *j* neighboring to the particle *i* is given by the following equation. Summing the heat transfer amount from the surrounding particles, the average value divided by N_i is to heat transfer amount Q_{ij}.

$$Q_{ij} = \frac{k}{N_i}\sum_{j=1}^{N_i}(t_j - t_i)dt \quad (i \neq j) \tag{6}$$

3.1.4. All Heat Transfer Amount to Particle

Equation (4), (5), and (6), the heat transfer amount Q_i is represented by the following equation are each in total.

$$Q_i = Q_{ia} + Q_{is} + Q_{ij} \tag{7}$$

The value of the thermal conductivity *k* varies with materials. In this model, the thermal conductivity of water is set to *k* = 0.601 [5].

3.2. Algorithm of Heat Transfer and Evaporation

It shows an algorithm of heat transfer and evaporation of water droplets using a previous section of the proposed model in Step 1 to 8. The temperature of the air and the temperature of the solid surface are always assumed to be constant. In addition, we will not take into account to wind, humidity, atmospheric pressure, and the convection of internal water droplets.

Step 1: To set the initial temperature of the particles and air. Usually, to set the initial temperature of the particles lower than that of air.
Step 2: Heat transfer occurs from the air to particles in the vicinity of the interface of the water droplet. By the equation (4), to calculate the heat transfer amount Q_{ia} between the air and the particle *i*.
Step 3: Heat transfer occurs from the solid surface to particles contacting to the solid surface. By the equation (5), to calculate the heat transfer amount Q_{is} between the solid surface and the particle *i*.
Step 4: When particles *j* neighboring to the particle *i* of the high temperature than the temperature of the particle *i*, that is, when $t_i < t_j$, heat transfer occurs from particle *j* to particle *i*. This is calculating by equation (6).
Step 5: The combined value of the Step 2 to 4, is added to the temperature t_i of particle *i* as the heat transfer amount Q_i.
Step 6: Particles reaches a certain temperature as evaporated to erase it.
Step 7: By part of the particle has been erased, force relations which had been working between the particles of the surrounding are changed.
Step 8: Until all the particles are eliminated, repeat Step 2 to 6.

By the process described above, how the water droplets evaporate is depicted.

3.3. Water Droplet Deformation by Evaporation

In this model, evaporation is depicted by erase the particles. In the SPH method, the movement of the particles are determined by the interaction of forces between the particles. Therefore, by the particle has been erased, force relations which had been working between the particles of the surrounding is changed. As a result, the water droplet model is deformed.

By evaporation, the volume of the droplets gradually decreases. At this time, its interface moves inward direction of the water droplets. The proposed method is applying to Sato's method [3]. Therefore, even if water droplet is moving themselves, without collapsing the water droplets likeness, the shape of a water drop is deformable.

4. RESULT

Using our proposed model, the result that depicts the state of the water droplets evaporate shown in Fig. 9.

Fig 9 (a) images displayed pseudo color temperature of the particles. Fig. 9 (b) shows the result was rendered using POV-Ray [6], generated polygons by Marching Cubes method [7] from the position of the particles.

It can be seen that is made depiction that the state of the evaporation of water droplets and its volume be gradually smaller.

5. CONCLUSION

In this study, we proposed a simple method to depict the evaporation by erasing the particles which reaches a certain temperature, to set parameters of the temperatures to the particles forming the water droplets, and consider the heat transfer in inside and outside of the water droplets. Previously it was depicted only the liquid state, to develop conventional interfacial tension and dynamic contact angle model, not only the movement of water droplets, was to be represented a phase transition over time.

In the future, the effect of the solid surface scratches and irregularities give the contact angle of a water droplet is also taken into consideration, so that the shape change different depending on the material it is possible to depict, overlaying the study.

11. REFERENCES

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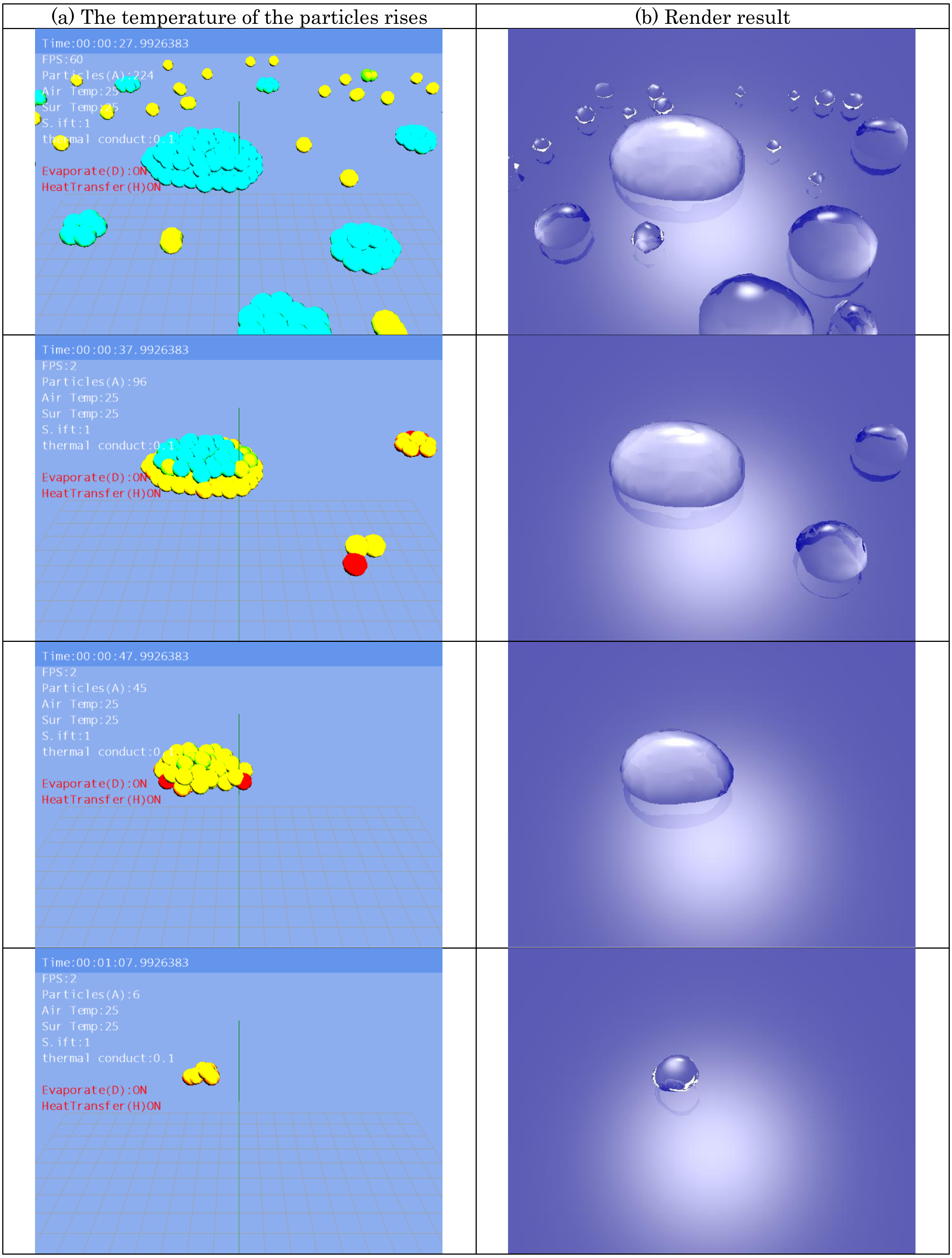

(a) The temperature of the particles rises

(b) Render result

```
Time:00:00:27.9926383
FPS:60
Particles(A):224
Air Temp:25
Sur Temp:25
S.ift:1
thermal conduct:0.1

Evaporate(D):ON
HeatTransfer(H)ON
```

```
Time:00:00:37.9926383
FPS:2
Particles(A):96
Air Temp:25
Sur Temp:25
S.ift:1
thermal conduct:0.1

Evaporate(D):ON
HeatTransfer(H)ON
```

```
Time:00:00:47.9926383
FPS:2
Particles(A):45
Air Temp:25
Sur Temp:25
S.ift:1
thermal conduct:0.1

Evaporate(D):ON
HeatTransfer(H)ON
```

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Time:00:01:07.9926383
FPS:2
Particles(A):6
Air Temp:25
Sur Temp:25
S.ift:1
thermal conduct:0.1

Evaporate(D):ON
HeatTransfer(H)ON
```

Fig. 9 Representation of evaporating water drops